Lecture 8. ***Functions are fixed-sign, constant-sign and alternating-sign***

We need the following information: fixed-sign, constant-sign and alternating-sign functions.

Let us consider a real n-dimension space R^n; a vector, belonging to it, will be $x \in R^n$.

Let the nonlinear system be described by the equation of general form:

$$\dot{x} = f(x),$$

The initial conditions are known: *f(0)=0.*

Let us introduce a scalar function *V(x)* of the vector argument. In space R^n we will investigate some area *D*, which obligatory includes the beginning of coordinates. Area "*D*" is an area of normal functioning of ACS.

Definition 1. Function $V(x)$ is one of fixed sign in area *D* if in this area it satisfies to the following conditions:

$$\begin{cases} V(x) \neq 0, \ if \ \ x \neq 0, \\ V(x) = 0, \ if \ \ x = 0. \end{cases}$$

Definition 2. Function $V(x)$ is one of fixed positive sign in area *D*, if in this area it satisfies to the following conditions:

$$\begin{cases} V(x) > 0, \ if \ \ x \neq 0, \\ V(x) = 0, \ if \ \ x = 0. \end{cases}$$

Definition 3. Function $V(x)$ is one of fixed negative sign in area *D*, if in this area it satisfies to the following conditions:

$$\begin{cases} V(x) < 0, \ if \ \ x \neq 0, \\ V(x) = 0, \ if \ \ x = 0. \end{cases}$$

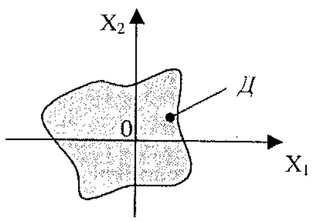


Fig.1. Area of normal functioning of the system

Example 6.6.

Let's $x \in R^2, V(x) = 2x_1^2 + 5x_2^2$ (fig. 2a)

Example 6.7.

Let's $x \in R^2, V(x) = -2x_1^2 - 5x_2^2$ (fig. 2b)

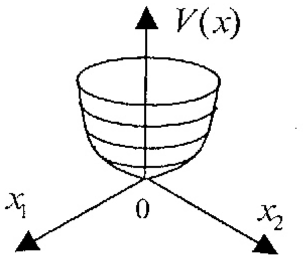


Fig. 2a. Function of fixed positive sign

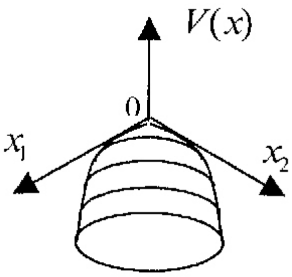


Fig. 2b. Function of fixed negative sign

Definition 4. Function *V(x)* is one of constant sign in area *D*, if in this area it satisfies to the following conditions:

$$\begin{cases} V(x) \neq 0, \text{ } if \ \ x \neq 0, \\ V(x) = 0, \text{ } if \ \ x = 0. \end{cases}$$

Definition 5. Function *V(x)* is one of positive constant sign in area *D*, if in this area it satisfies to the following conditions:

$$\begin{cases} V(x) \geq 0, \text{ } if \ \ x \neq 0, \\ V(x) = 0, \text{ } if \ \ x = 0. \end{cases}$$

Definition 6. Function *V(x)* is one of negative constant sign in area *D,* if in this area it satisfies to the following conditions:

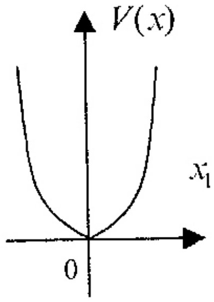
$$\begin{cases} V(x) \leq 0, \text{ } if \ \ x \neq 0, \\ V(x) = 0, \text{ } if \ \ x = 0. \end{cases}$$

Example 6.8.

Let's $x \in R^1, V(x) = 2x^2$ (fig. 2c)

Example 6.9.

Let's $x \in R^2, V(x) = x_1^2 + x_2^2$ (fig. 2d)



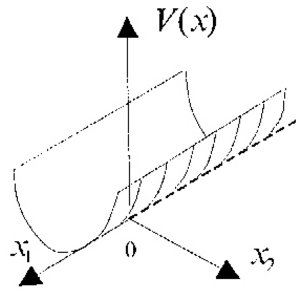


Fig. 2c. Function of fixed positive sign Fig. 2d. Function of constant positive sign

Example 6.10. Let $x \in R^n$. Functions are *V(x)* = x^T, *x* – fixed positive sign;
V(x) = -x^T, *x* – fixed negative sign.

Example 6.11. Let $x \in R^3$ (fig. 2e.)

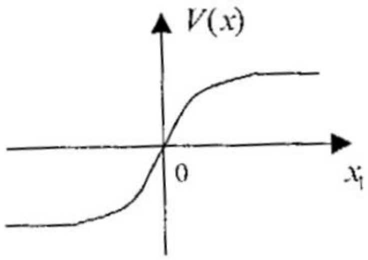


Fig. 2e. Alternating-sign function

Definition 7. Function *V(x)* is one of alternating sign in area *D*, if in this area it can be of both signs *V(x), V(x)* > 0 and *V(x)* < 0, in case if $x \neq 0$.